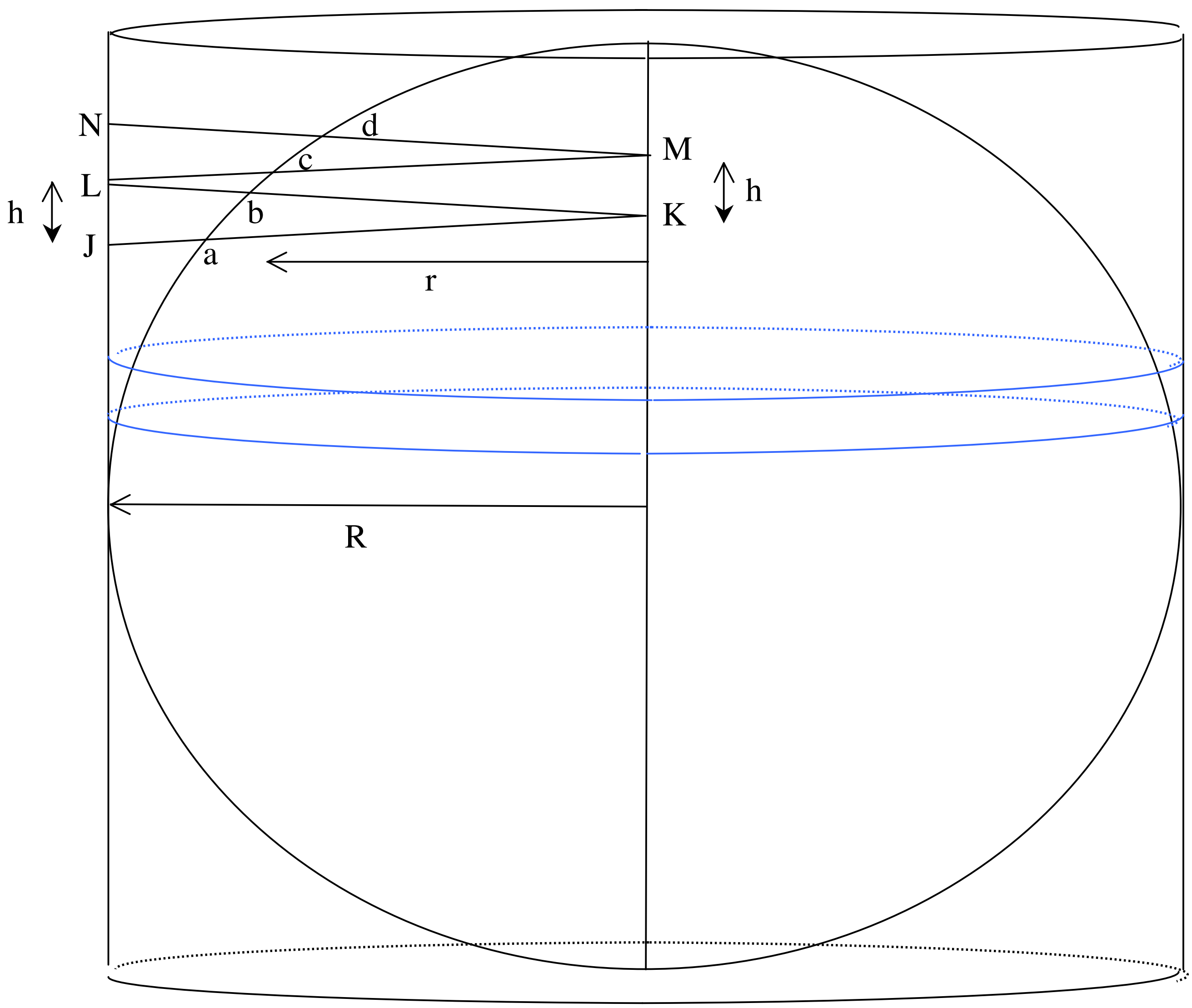
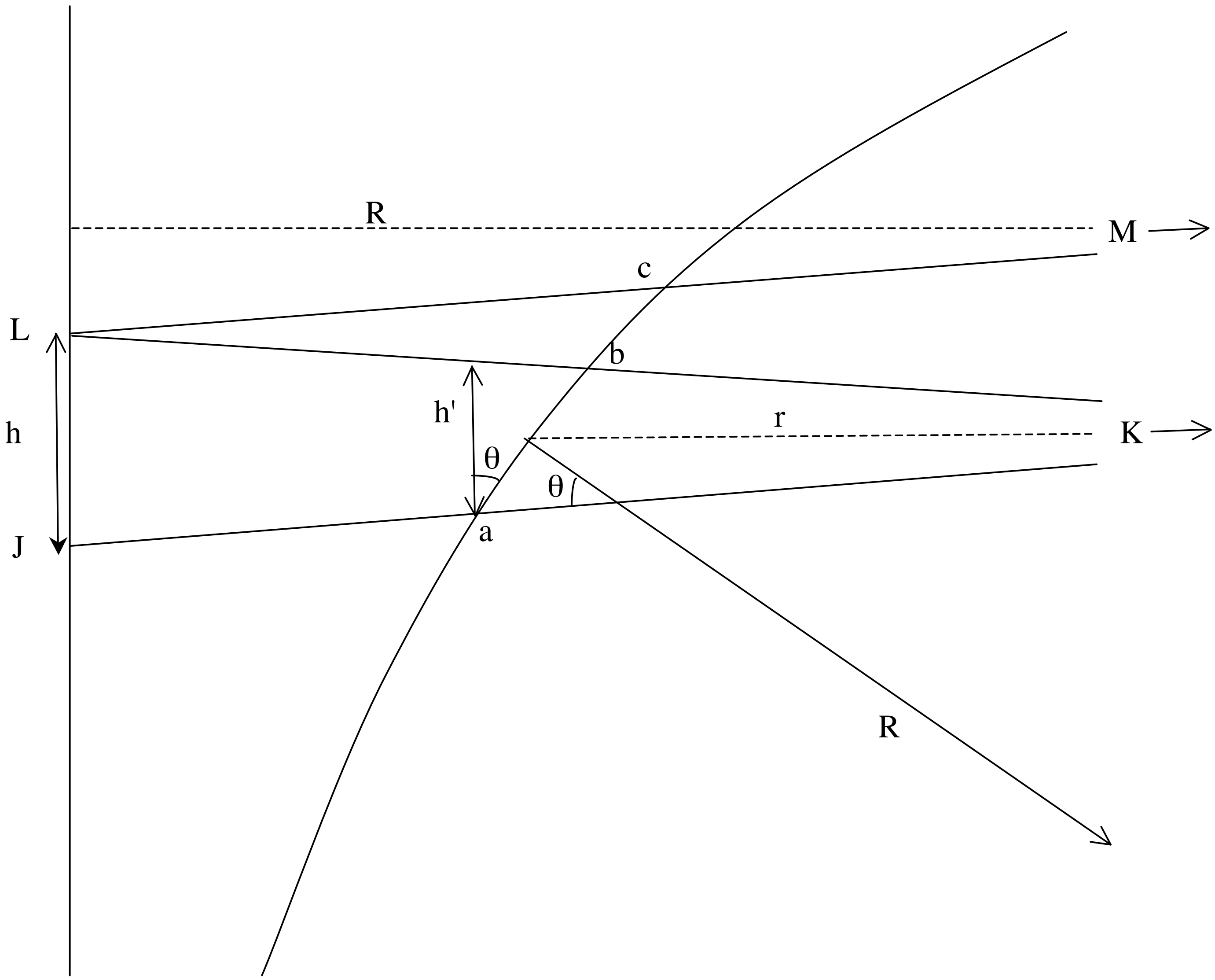
When comparing the surface area of the cylindar wall to that of the spheree, we first divide the wall into equally spaced bands with boundaries labeled J, L, M etc., for example. An example band is shown in blue. The idea is to see how those bands map onto the sphere's surface.
Next we vizualize triangles such as JKM and LMN where the heights of K and M (located on the sphere's diameter) are halfway between the heights of J & L and L & N, respectively.
Let R = the radius of the cylindar and sphere, r = the distance from the diameter to the sphere surface, and h = the distance from K to M = distance from J to L etc.

If we choose very small divisions along the cylindar wall, then it is clear that the band contructed at the sphere's equater will have the same surface area as the sphere for the same vertical range.
However, as we examine bands further up, we address three things.
1) It is not clear whether distances a -> b are still h.
2) In projecting the band bounded by J and L on to the sphere (for example), it is clear that there is more area available in the band than is needed to cover the same vertical range on the sphere (i.e. $2\pi R > 2\pi r$).
This effect is more pronounced as we go higher vertically with our bands.
3) As we go higher up in bands, gaps between coverage on the sphere also become more noticeable.
Note the gap between points b and c on the sphere. These gaps make sense because we cannot expect a line of height 2R (cylindar wall) to entirely wrap around a hemisphere that spans πR in measure.



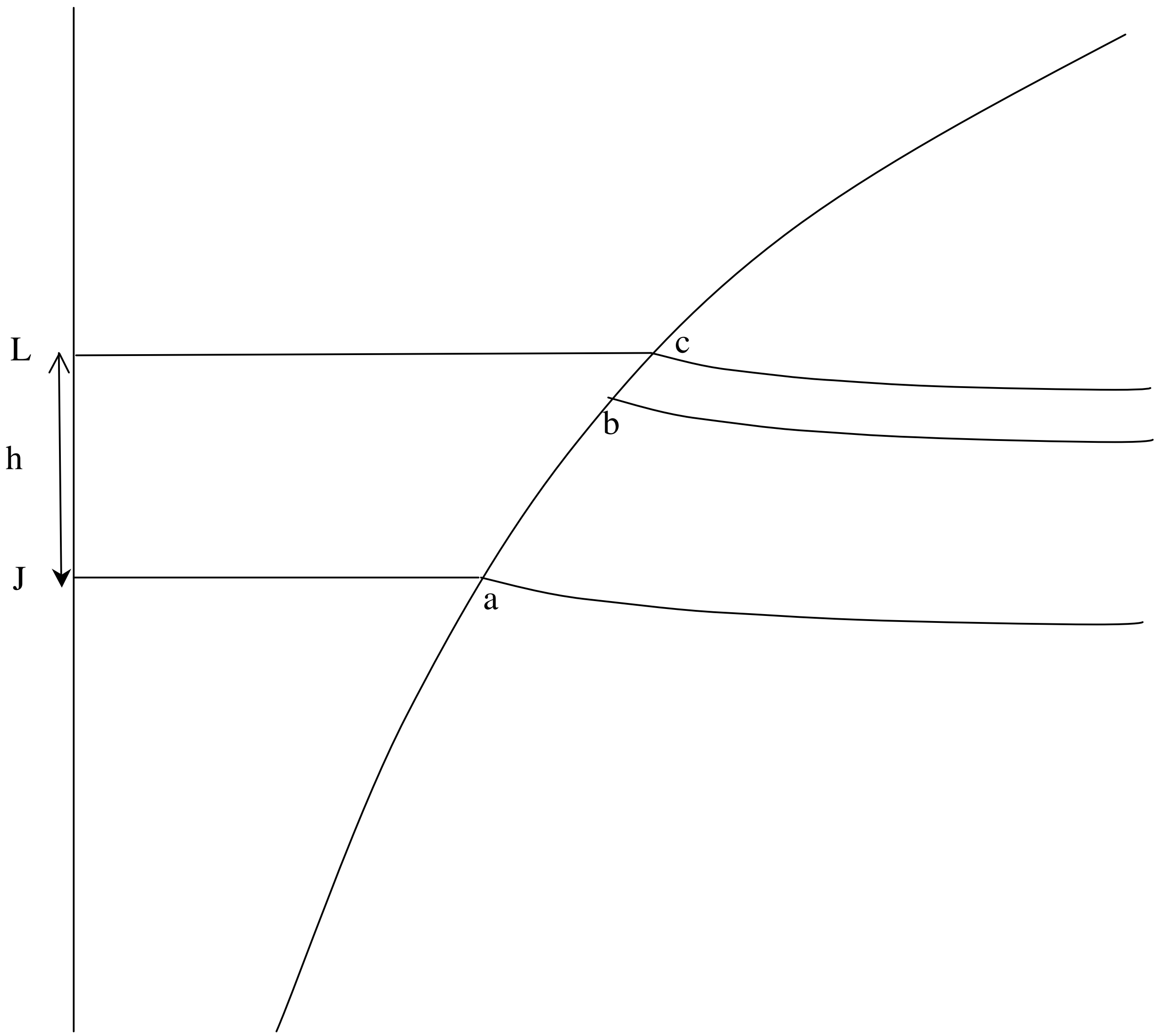
For number 1 above we consider the following blow-up diagram (line segments on the right are cut off). For small values of h, we may consider r as a good approximation for the distance from points J and L to point K. Thus, h' = hr/R and the distance a->b is approximately h'/cos(θ). For small h, line JK is nearly horizontal and cos(θ) is aprroximately r/R. Therefore the distance a->b is simply h.
The gap represented by b->c may be calculated in a similar fashion (using triangle KLM) as [h(R-r)/R]/cos(θ) = h(R-r)/r



The excess area for a given band is 2π(R-r)h and the area needed to cover the adjacent gap is 2πr[h(R-r)/r] = 2πh(R-r).
Thus, it turns out that the excess area for each band is exactly the amount of area needed to cover the adjacent gap.

In retrospect one can see that projecting the cylindar onto the sphere using triangles conveniently matched the verticle distance covered for both the geometric shapes. However, one could have dispensed with the triangles and got the same result.

In the following figure we project the cylindar bands onto the sphere straight across horizontally.



For each height element h on the cylindar we see that it will only cover part of the distance a -> c along the sphere's surface. If we let a -> b = h, then b -> c represents the gap not covered by the projection. From the work above we know that a -> c = hR/r, thus b -> c = h(R-r)/r. The surface area of the gap is then $2\pi h(R-r)$. The excess area offered by the cylindar for the band bounded by a and b is $2\pi h(R-r)$. Thus, the surface areas of the cylindar wall and the sphere are the same.